OPTICAL TECHNOLOGY DIVISION NIST

Diffraction effects on total solar irradiance measurements

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Contents		**Conclusions**
Outline of method to calculate diffraction effects (corrections) (4)	→	**Theoretical diffraction effects are easy to estimate.**
Diffraction effects for instruments (1)	→	**Diffraction effects here are small, can be known, and should be taken into account.**
Diffraction correction uncertainty analysis (5)*	→	**We probably know diffraction effects to within a few percent of the effects. Measurement uncertainties limit testing of theory.**
Conclusions (this slide)		

*Done in collaboration with R.U. Datla and R. Kacker (NIST Statistical Engineering Division)

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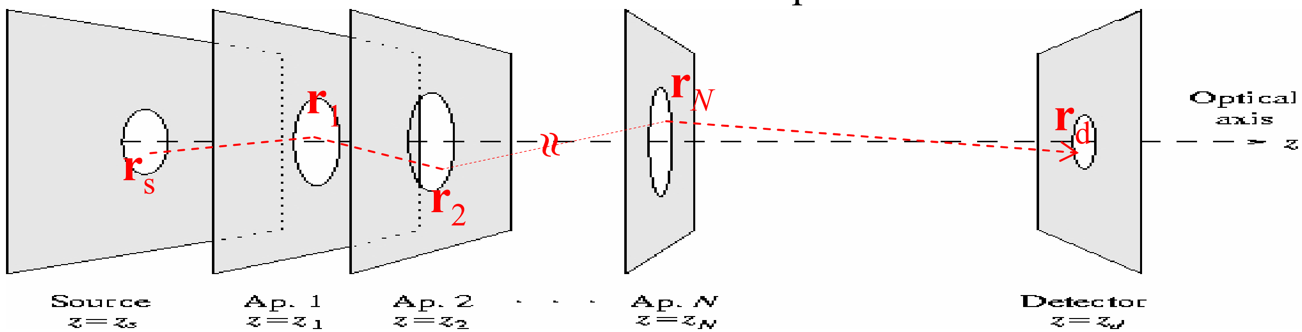
Diffraction effects on an optical measurement:

$$\Phi = EA \rightarrow \Phi = EAF$$

power (Φ), TSI (E), aperture area (A), Diffraction factor (F)

$$E = \frac{\Phi}{AF}$$

Consider paths summed over in Kirchhoff theory:



SUN | View-limiting/defining apertures | ACR Entrance

TSI monitoring instrument

Path length (in the Fresnel, paraxial approximation)

$$L(\{\mathbf{r}_\mu\}) = z_\mathrm{d} - z_\mathrm{s} + \frac{(x_1 - x_\mathrm{s})^2 + (y_1 - y_\mathrm{s})^2}{2(z_1 - z_\mathrm{s})} + \cdots + \frac{(x_\mathrm{d} - x_N)^2 + (y_\mathrm{d} - y_N)^2}{2(z_\mathrm{d} - z_N)} + \delta L_\mathrm{foc}(\{\mathbf{r}_\mu\})$$

Iterated Kirchhoff formula gives

$$u(k, \mathbf{r}_\mathrm{s}, \mathbf{r}_\mathrm{d}) = \frac{u_0}{(\mathrm{i}\lambda)^N} \int_{\mathrm{Ap1}\cdots\mathrm{Ap}N} \mathrm{d}^2\mathbf{r}_1 \cdots \mathrm{d}^2\mathbf{r}_N G(\mathbf{r}_\mathrm{s}, \mathbf{r}_1; k) \cdots G(\mathbf{r}_N, \mathbf{r}_\mathrm{d}; k) \mathrm{e}^{\mathrm{i}kL(\{\mathbf{r}_\mu\})}$$

free-space, two-point Green's function

$$G(\mathbf{r}, \mathbf{r}'; k) = \frac{\exp(\mathrm{i}k\,|\,\mathbf{r} - \mathbf{r}'\,|)}{|\,\mathbf{r} - \mathbf{r}'\,|}$$

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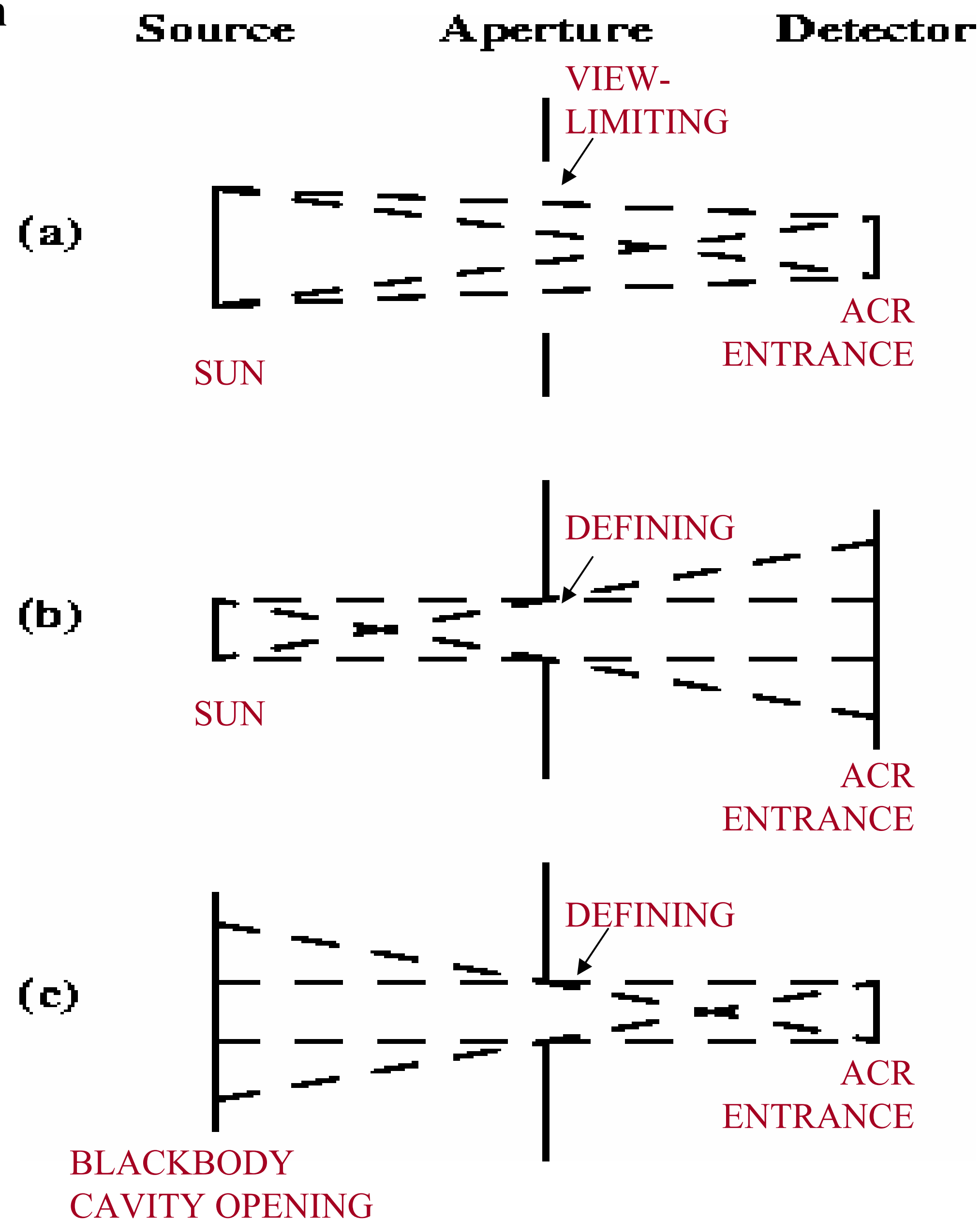
Simplified diffraction calculation
Source-aperture-detector (SAD) Problem

Three main types of SAD geometries:

Overfilled detector:
PMO6, VIRGO, SOVIM, DIARAD, ACRIM, ERBE

Underfilled detector:
SORCE

Undersampled source:
Blackbody calibration



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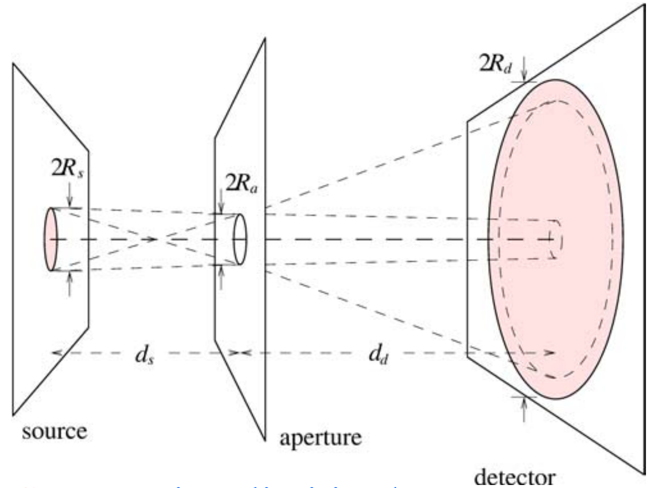
SAD Problem: Diffraction effects on spectral power

Case of a symmetric optical system (Shirley, JOSA A, 2004)

As shown, the detector is underfilled (e.g., SORCE). If the detector perimeter were within the smaller dashed circle, it would be overfilled (e.g., ERBE, ACRIM, PMO6, VIRGO, SOVIM, DIARAD).



SUN

view-limiting/ or defining aperture

ACR entrance

From the geometry, introduce parameters,

$$u = (2\pi R_a^2/\lambda)(1/d_s + 1/d_d - 1/f),$$
$$v_s = (2\pi/\lambda)(R_s R_a/d_s),$$
$$v_d = (2\pi/\lambda)(R_d R_a/d_d),$$
$$v_0 = \max(v_s, v_d),$$
$$\sigma = \min(v_s, v_d)/\max(v_s, v_d),$$
$$w = \min(u/v)/\max(u/v)$$
$$g = (w + 1/w)/2$$

From Wolf, we have the result, for an axial point source,

$$Y_n(u,v) = \sum_{s=0}^{\infty}(-1)^s (n+2s)(v/u)^{n+2s} J_{n+2s}(v)$$
$$Q_{2s}(v) = \sum_{p=0}^{2s}(-1)^p [J_p(v) J_{2s-p}(v) + J_{p+1}(v) J_{2s+1-p}(v)]$$
$$L_B(v,w) = \sum_{s=0}^{\infty}(-1)^s w^{2s} Q_{2s}(v)/(2s+1)$$
$$L_X(v,w) = (4w/v)[Y_1(v/w, v)\cos(gv) + Y_2(v/w, v)\sin(gv)]$$
$$L(u,v) = w^2[1 + L_B(v,w)] - L_X(v,w), v < u$$
$$L(u,v) = 1 - L_B(v,w), v > u$$

We have asymptotic results for $L_B(v,w)$ and $L_X(v,w)$ at large v.

fraction of flux incident on aperture that reaches detector

Diffraction effects on flux reaching detector:

$$\Phi_\lambda(\lambda) = C\int_{-1}^{1} dx (1+\sigma x)^{-1}\{(1-x^2)[(2+\sigma x)^2 - \sigma^2]\}^{1/2} L(u, v_0(1+\sigma x)) L_\lambda(\lambda)$$

$v = v_0(1+\sigma x)$

$$C = 4\pi^3 R_a^4 R_s^2 R_d^2 / [d_s^2 d_d^2 (\lambda v_0)^2]$$

$$F(\lambda) = \frac{\Phi_\lambda(\lambda)}{\Phi^{\text{ideal}}(\lambda)}$$

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SAD Problem: Diffraction effects on total power for a thermal source

Case of a symmetric optical system (Shirley, JOSA A, 2004)

For a thermal source, we have, with $v=v_0(1+\sigma x)$,

$$\Phi = C\int_{-1}^{1} dx(1+\sigma x)^{-1}\{(1-x^2)[(2+\sigma x)^2-\sigma^2]\}^{1/2}\int_0^\infty d\lambda\, L(u,v)\, L_\lambda(\lambda)$$

Introducing

small parameter

$$\alpha = v\lambda \qquad A=\frac{c_2}{\alpha T} \qquad F_X(A,w)=\int_0^\infty \frac{dv\, v^3}{\exp(Av)-1}L_X(v,w) \qquad F_B(A,w)=\int_0^\infty \frac{dv\, v^3}{\exp(Av)-1}L_B(v,w)$$

we have, for a [small] thermal source,

$$\int_0^\infty d\lambda\, L(u,v)\,L_\lambda(\lambda)=\frac{\varepsilon c_1}{\pi\alpha^4}\int_0^\infty \frac{dv\, v^3}{\exp(Av)-1}\{w^2[1+L_B(v,w)]-L_X(v,w)\} = \frac{\varepsilon c_1}{\pi\alpha^4}\left[\frac{6w^2\zeta(4)}{A^4}+w^2F_B(A,w)-F_X(A,w)\right].$$

or

$$\int_0^\infty d\lambda\, L(u,v)\,L_\lambda(\lambda)=\frac{\varepsilon c_1}{\pi\alpha^4}\int_0^\infty \frac{dv\, v^3}{\exp(Av)-1}[1-L_B(v,w)] = \frac{\varepsilon c_1}{\pi\alpha^4}\left[\frac{6\zeta(4)}{A^4}-F_B(A,w)\right]$$

Asymptotically, we have...

$$\frac{A^4F_X(A,w)}{6\zeta(4)} \sim -\left(\frac{16w^6+48w^8+16w^{10}}{\zeta(4)(1-w^2)^7}\right)A^4+O\left\{\exp\left[-\frac{\pi(w+1/w)}{A}\right]\right\}$$

$$\frac{A^4F_B(A,w)}{6\zeta(4)} \sim \left(\frac{2\zeta(3)}{3\pi\zeta(4)(1-w^2)}\right)A + \left(\frac{-1+20w^2+90w^4+20w^6-w^8}{24\pi\zeta(4)(1-w^2)^5}\right)A^3\log_e(A)$$
$$-\left(\frac{(2\gamma+6\log_e 2)(-3+60w^2+270w^4+60w^6-3w^8)-768(w^3+w^5)\log_e[(1+w)/(1-w)]+9-228w^2-1354w^4-228w^6+9w^8}{144\pi(1-w^2)^5}\right)A^3$$
$$-\left(\frac{4w^4+12w^6+4w^8}{\zeta(4)(1-w^2)^7}\right)A^4 + \left(\frac{3+8w^2-2412w^4-51912w^6-120750w^8-51912w^{10}-2412w^{12}+8w^{14}+3w^{16}}{1536\pi\zeta(4)(1-w^2)^9}\right)A^5\log_e A \quad + \quad O(A^5)$$

Diffraction effect

$$F(A,w)-1\approx\pm\frac{0.2357A}{1-w^2}$$

Rapidly Converging expansions

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Results for diffraction effects on TSI measurements:

NOTE: The factor $<F>$ describes the ratio of actual power to ideal power.
Therefore, raw measured power is corrected by dividing it by $<F>$.

$$E = \frac{\Phi}{A < F >}$$

For all radiometers, we have:

$$R_s = 6.75\times 10^{11}\ \text{mm},\ d_s = 1.5\times 10^{14}\ \text{mm}$$

Otherwise, diffraction effects are given as follows (assuming T=5900 K for the sun):

Radiom.	R_a (mm)	d_d (mm)	R_d (mm)	type	$<F>-1$
PMO6	4.25	95.4	2.5	a	+0.001280
VIRGO	4.25	98.5	2.5	a	+0.001323
SOVIM	4.8	98.5	2.5	a	+0.001025
DIARAD	6.52	144.	4.0015	a	+0.000833
ERBE	12.09	100.8	4.039	a	+0.000209
ACRIM					
Baf1	6.6548	150.4696	3.9878	a	+0.000828
Baf2	6.3119	76.3524	3.9878	a	+0.000466
Total					→ +0.001295
TIM	3.9894	101.6	7.62	b	-0.000418

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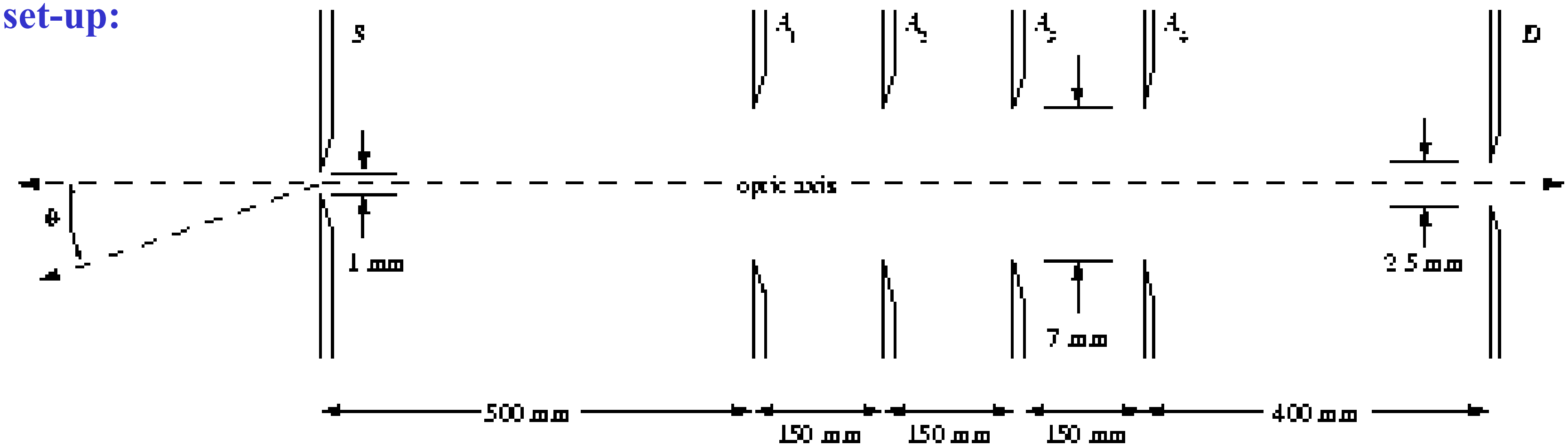
Estimating systematic uncertainties of diffraction calculations:

Test case: Optical set-up of Boivin [Appl. Opt. 17, 3233 (1976); tungsten lamp source, photomultiplier detector,areas defined apertures; Boivin investigated role of non-limiting apertures on throughput (a diffraction effects)].

Sample set-up:



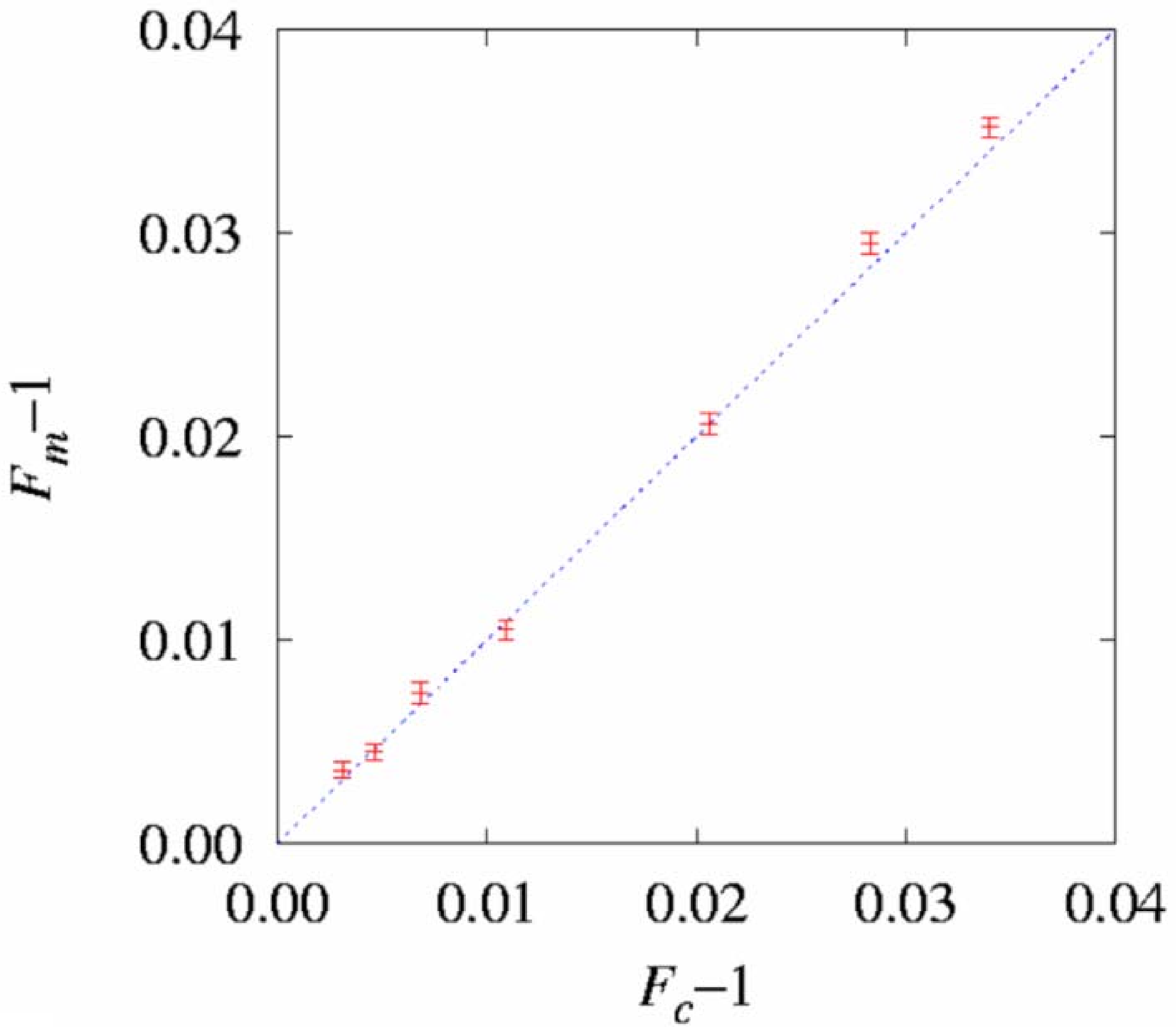
We use these data to test our diffraction model. In this case, we have

$$F_{\mathrm{eff}} - 1 = \frac{\int \mathrm{d}\lambda\, \Phi_{0,\lambda}(\lambda) g(\lambda)[F(\lambda) - 1]}{\int \mathrm{d}\lambda\, \Phi_{0,\lambda}(\lambda) g(\lambda)}$$

where $g(\lambda)$ accounts for spectral responsivity of detector.

Estimating systematic uncertainties of diffraction calculations:

Comparison of results in test case (k=1 meas. unc. shown):



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Analysis Method

Assume that *relative* uncertainty of theoretical diffraction effects is sought.
That is, we seek to estimate the typical difference of theoretical diffraction effects from actual diffraction effects. Suppose that we have

$$\{(F_{m,i}, F_{c,i}), i = 1, N\}$$

Define

$$Y_{m,i} = \log_e |F_{m,i} - 1|, \quad Y_{c,i} = \log_e |F_{c,i} - 1|, \quad D_i = Y_{m,i} - Y_{c,i}$$

Also have

$$< D > = N^{-1} \sum_{i=1}^{N} D_i$$

$$V(D) = (N-1)^{-1} \sum_{i=1}^{N} (D_i - < D >)^2$$

$<D>$ indicates a non-zero average error in Y_c.

$V(D)$ indicates typical achievable agreement between measurement and theory.

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Analysis Method, continued.

Given measurement uncertainties, it is easy to establish the variance,

$$V(Y_m) = \left\langle u^2(Y_m) \right\rangle$$

One can also calculate the data,

$$D_i = Y_{m,i} - Y_{c,i}$$

And from this relation, one may deduce

$$V(D) = V(Y_m) + V(Y_c)$$

THEREFORE, $u(Y_c)$ can then be deduced from

$$\left\langle u^2(Y_c) \right\rangle = V(Y_c) = V(D) - V(Y_m)$$

Results of comparison:

In our example, we have

$$V(D)=0.00435=(0.066)^2,$$

$$\langle D \rangle=0.034,$$

indicating no significant non-zero average error in the theory, and

$$V(Y_m)=0.00402,$$

Giving

$$V(Y_c)=0.000328=(0.018)^2,$$

or a 1.8 % *estimated* relative standard uncertainty of Y_c.